THE REGION OF DISCONTINUOUS SOLUTIONS OF VARIATIONAL PROBLEMS IN GAS DYNAMICS

(OBLAST' RAZRYVNYKH RESHENII VARIATSIONNYKH ZADACH GAZOVOI DINAMIKI)

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In [1] a solution was found of the variational problem of determining the shape ab of a body of revolution having minimum drag (Fig. 1) with the presence of an isentropic discontinuity at the point h, when the Mach wave ac of the oncoming supersonic stream is given together with the location of the points a and b. The waves of the fan ahk focus at

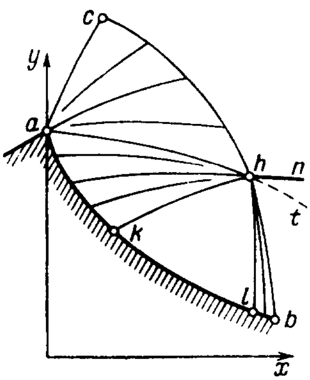

Fig. 1.

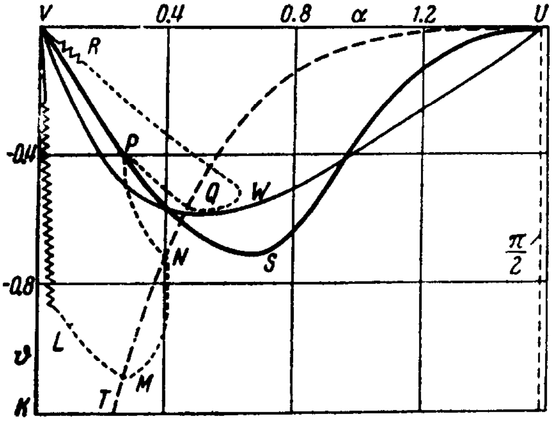

Fig. 2.

the point h. From point h there emerges a shock wave hn, a line of contact discontinuity ht and a fan of Mach waves lhb. The discontinuity in the Mach angle α and in ϑ, the angle of inclination of the velocity with the x-axis, is determined on the line cb at the point h by two transcendental equations.

Values at the point h depend on the direction of approach to that point. The existence of a discontinuity is determined by the following conditions (where the second subscript indicates the direction of approach):

$$\alpha_{hk} \leqslant \pi/2, \qquad \vartheta_{hb} \geqslant G(\alpha_{hb}), \qquad \vartheta_{hb} \geqslant H(\alpha_{hb}), \qquad \vartheta_{hc} \leqslant H(\alpha_{hc}) \tag{1}$$

$$G(\alpha) = -\tan^{-1}\frac{\sin 2\alpha}{2\varkappa - 1 - \cos 2\alpha}, \qquad H(\alpha) = -\tan^{-1}\frac{(1 + \cos 2\alpha)\sin 2\alpha}{\varkappa + \cos^2 2\alpha}$$

The first condition indicates that subsonic speeds are inadmissible, the second indicates that point h of the line bh belongs to the region of the solution without shocks, and the last two determine the location of points with coordinates α_{hc}, ϑ_{hc} and α_{hb}, ϑ_{hb} such that the minimum of the drag is attained. Furthermore, the condition of the existence of the indicated flow configuration in the vicinity of the point h must be satisfied.

In view of the complexity of the equations determining the discontinuity at point h, and of the boundary conditions given for the desired region, the solution was found numerically. The calculations were carried out for an adiabatic exponent $\kappa = 1.4$. In the region of variation of α and ϑ the roots of the discontinuity equations were determined and the realization of the given conditions was verified. In Fig. 2 the line *VWU* shows the relation $\vartheta = G(\alpha)$ and the line *VSU* the relation $\vartheta = H(\alpha)$. Admissible α_{hc} and ϑ_{hc} belong to the region *VLMNPV*. The corresponding α_{hb} and ϑ_{hb} lie in the region *VRQPV*. The line *PNMLV* is the boundary of existence of the given configuration at the point h. The curve *LV* lies on the limit of the sonic line.

If flow in a nozzle is considered, the quantity ϑ is to be replaced by $-\vartheta$. The region of expanding flow of the type *cah* is bounded by the broken line *KVUT*, where *UT* represents a characteristic of the Prandtl-Meyer flow. The solution found here includes practically the entire region of variation of the parameters of a nozzle.

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